Performance Evaluation of Data Transmission over Multicarrier CDMA Systems

Feng-Tsun Chien*, Chien-Hwa Hwang*, Chang-Su Kim+ and C.-C. Jay Kuo*

*Department of Electrical Engineering and Integrated Media Systems Center
University of Southern California, USA

+School of Electrical Engineering, Seoul National University, Korea

ABSTRACT

The performances of direct sequence-code division multiple access (DS-CDMA), multicarrier-CDMA (MC-CDMA) and multicarrier-direct sequence-CDMA (MC-DS-CDMA) systems under different channel conditions are compared in this work. In a frequency-selective slowly fading channel, MC-CDMA and MC-DS-CDMA outperform DS-CDMA, since the former two systems partition the frequency band into sub-channels, each of which has a nearly constant frequency response. Thus, MC-CDMA and MC-DS-CDMA do not suffer much from the multipath effect. The performance of MC-CDMA and MC-DS-CDMA can be further differentiated in severe fading conditions. In a frequency-selective fast fading channel, the larger spreading ratio of MC-DS-CDMA in the time domain prevents the chip duration of a sub-carrier from being longer than the channel coherence time. Hence, the sub-carrier orthogonality is maintained in MC-DS-CDMA, leading to its better performance in this case.

Keywords: Code division multiple access (CDMA), direct sequence-CDMA (DS-CDMA), multicarrier-CDMA (MC-CDMA), multicarrier-direct sequence-CDMA (MC-DS-CDMA), frequency-selective, time-selective, Rayleigh fading channel.

1. INTRODUCTION

The direct sequence-code division multiple access (DS-CDMA) system has received much attention recently due to its several advantages. In a CDMA system, new users are allowed to access the system as long as the quality of wireless links is maintained at a certain level. To accommodate more users, powerful channel protection schemes are employed so that the system can be operated under higher interference levels. In this way, CDMA is more flexible in its capacity. Moreover, due to the use of the spread spectrum technology, CDMA has additional advantages such as jamming rejection and multipath mitigation.

The spreading process in CDMA can alleviate the multipath effect using the fact that the spreading sequences are almost uncorrelated to each other regardless of respective delays. It follows that the contribution of other paths to the decision statistic is small. However, even though the contributed amount is small, it is not negligible, especially under some environments, *e.g.* fading channels, where the low correlation property can no longer be guaranteed. The interference resulting from the multipath effect, called multipath inter-symbol interference (ISI), deteriorates the performance of a CDMA system. Since the multipath problem in the time domain is a dual of the frequency-selective fading in the frequency domain, the multipath effect can be mitigated if the signal bandwidth is reduced to be much smaller than the channel coherence bandwidth.

The multicarrier modulation techniques have been demonstrated as an effective approach in combating the hostile frequency-selective multipath fading channel [1,2]. The data sequence is segmented into blocks, and transmitted over a number of parallel sub-carriers, with a reduced symbol rate on each of them and together to yield the same total data rate as that of the single-carrier system. If the number of sub-carriers is large enough, the whole bandwidth is subdivided into a number of sub-channels with each having roughly a constant frequency response. Thus, the signal transmitted over each sub-channel encounters a frequency-flat fading channel such that the multipath effect diminishes. However, since the symbol rate on each sub-carrier is reduced, the symbol duration may be longer than the coherence time. In consequence, the multicarrier system is vulnerable to the fading effect in the time domain.

As mentioned above, a CDMA system is effective in suppressing the multipath effect by the orthogonality property of spreading sequences, but still suffers from the frequency-selective fading. On the other hand, a multicarrier transmission system subdivides the whole bandwidth into frequency-nonselective channels, but is apt to be affected

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by the time-selective fading. It is desirable if the advantages of these two concepts are combined together. In this way, the CDMA frequency band is partitioned into sub-channels with constant frequency responses, thus the frequency-selective fading problem being removed. Meanwhile, the time domain fading problem of the multicarrier system is relieved due to the spreading mechanism of CDMA.

To inherit the advantages from both CDMA and multicarrier modulation techniques, several multicarrier CDMA systems have been proposed and investigated during the past decade. The multicarrier-CDMA (MC-CDMA) system was first proposed in [3], where each data symbol is transmitted over a set of parallel sub-carriers, with each being modulated by one chip of the spreading sequence. The concept of this system originates from the combination of spreading and multicarrier transmission in that the spreading process is carried out in the frequency domain. By modifying the mechanism of the spreading process, other multicarrier systems were also presented [4,5]. In [4], Kondo and Milstein proposed a multicarrier-direct sequence-CDMA (MC-DS-CDMA) system, where the spreading process is executed in both frequency and time domains. Thus, it exhibits the capability in both suppressing narrowband interference and mitigating the frequency-selective fading effect. At last, various sophisticated multiuser detection schemes applied to multicarrier CDMA systems can be found in recent publications [6,7,8,9,10].

In this paper, we attempt to analyze and compare the performance distinction among single-carrier DS-CDMA, MC-CDMA and MC-DS-CDMA systems under different channel conditions. Let us clarify some ambiguous terminologies. Throughout the paper, we use multicarrier CDMA to refer to both MC-CDMA and MC-DS-CDMA systems. The MC-CDMA is used, when the scheme proposed in [3] is mentioned. At last, the single-carrier CDMA represents DS-CDMA. A fair comparison of three systems is done with an equal composite spreading ratio N (the product of the spreading ratios in frequency and time domains) [6]. It was studied in [11,12] that multicarrier CDMA systems perform better than a single-carrier DS-CDMA system in a slowly time-varying frequency-selective fading environment. In this paper, the performance of the two multicarrier CDMA systems will be differentiated. We will investigate the environment subject to fast time-varying fading such that the fading coefficient changes within one symbol duration of a sub-carrier. Performance degradation will occur due to the loss of the orthogonality between sub-carriers, which results in the inter-carrier interference (ICI). In such a channel, MC-DS-CDMA is preferable because the spreading process in the time domain reduces the chip duration of a sub-carrier.

The rest of the paper is organized as follows. The system model is introduced in Section 2. In Section 3, the maximal ratio combining (MRC) technique is described, and the performance of multicarrier CDMA systems are analyzed. In Section 4, comparison of simulation results for single-carrier CDMA and multicarrier CDMA systems under different channel environments are presented. Finally, we provide some concluding remarks in Section 5.

2. SYSTEM MODEL

2.1. Transmitter Model

Consider the uplink of a synchronous multicarrier CDMA system with M sub-carriers. The ith data symbol of user k, $b_k(i)$, with duration T_b is duplicated to each of the M output branches of a serial-to-parallel (S/P) converter. At each branch, symbol $b_k(i)$ is spread by the corresponding signature waveform, modulated with its respective sub-carrier, and then transmitted over a wireless channel. Stated mathematically, for both MC-CDMA and MC-DS-CDMA systems, the transmitted signal of user k in complex analytic form can be expressed as [6]

$$s_k(t) = \sum_{m=0}^{M-1} \sqrt{\frac{2P_k}{M}} \left(\sum_{i=-\infty}^{\infty} b_k(i) c_{k,m}(t - iT_b) \right) e^{j\omega_m t}, \tag{1}$$

where P_k is the average transmitted power of user k, ω_m is the modulation radian frequency at the mth sub-carrier, and $c_{k,m}(t)$ is the normalized spreading waveform of user k at the mth sub-carrier. Note that the expressions of $c_{k,m}(t)$ for MC-CDMA and MC-DS-CDMA are not the same, *i.e.* they have different spreading mechanism. The transmitter block diagrams for MC-CDMA and MC-DS-CDMA systems are shown in Figure 1.

In an MC-CDMA system, each user is assigned with a single signature sequence [3]. On each sub-carrier, the data symbol is multiplied by one chip of the signature sequence. Specifically, the $c_{k,m}(t)$ in (1) is given by

$$c_{k,m}(t) = c_k^{(m)} p(t), \quad m = 0, 1, .., M-1, \tag{2}$$

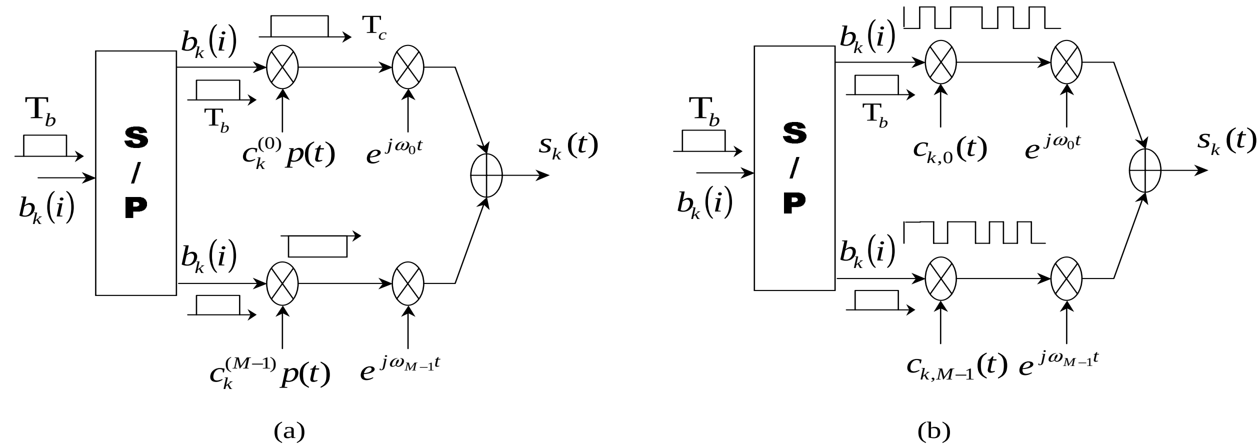

Figure 1. The transmitter block diagram of user k for multicarrier CDMA systems: (a) MC-CDMA, and (b) MC-DS-CDMA.

where $c_k^{(m)}$ is the mth chip of user k's signature sequence, and $p(t)$ denotes the chip pulse shaping waveform, which is assumed to be a unit amplitude rectangular waveform with time span T_b in this paper for simplicity. Note that, in MC-CDMA, the length (or the number of chips) of a signature sequence is equal to the number of sub-carriers. Moreover, it can be seen from Figure 1(a) that the spreading process is carried out in the frequency domain.

In an MC-DS-CDMA system, a set of M distinct signature sequences are assigned to a user, and each of them is used at a sub-carrier [10][13]. In MC-DS-CDMA, the spreading waveform $c_{k,m}(t)$ in (1) is expressed as

$$c_{k,m}(t) = \sum_{j=0}^{N_c-1} c_{k,m}^{(j)} p(t - jT_b/N_c), \tag{3}$$

where $c_{k,m}^{(j)}$ is the jth chip of user k's signature sequence applied at the mth sub-carriers, N_c is the length of a signature sequence, and the time span of $p(t)$ here is equal to T_b/N_c. In contrast with the case in MC-CDMA where the number of chips is equal to M, the choice of N_c in MC-DS-CDMA is more flexible. If $N_c > 1$, besides the frequency domain spreading resulting from the bandwidth subdivision, the spreading process is also carried out in the time domain. When $N_c = 1$, the MC-DS-CDMA is degenerated to an MC-CDMA system.

The relation between MC-CDMA, MC-DS-CDMA and single-carrier DS-CDMA can be demonstrated by defining a composite spreading ratio N as the product of the number M of sub-carriers (the spreading ratio of the process carried out in the frequency domain) and the number of chips N_c per bit (spreading ratio of the process carried out in time domain), *i.e.* $N = MN_c$. The single-carrier DS-CDMA and MC-CDMA correspond to two extreme cases of $M = 1$ and $N_c = 1$, representing no spreading executed in the frequency and the time domains, respectively, while MC-DS-CDMA lies in between. Since the composite spreading ratio is the effective spreading gain earned from both domains, when conducting the performance evaluation, we have to assign the same ratio N to different systems for fair comparison. The MC-DS-CDMA can be regarded as a hybrid of MC-CDMA and DS-CDMA, which allocates the spreading gain between time and frequency domains.

2.2. Channel Model

In this paper, we assume a wide sense stationary uncorrelated scattering (WSSUS) fading channel model. Depending on their bandwidths and symbol durations, two signals transmitted in the same wireless environment may suffer from different channel effects. The channels experienced by single-carrier DS-CDMA and multicarrier CDMA systems are discussed, respectively, in this subsection.

Consider a DS-CDMA system over a frequency-selective slowly fading channel. In such a case, we have the signal bandwidth $W \gg (\Delta f)_c$ and time duration $T_b \ll (\Delta t)_c$, where $(\Delta f)_c$ and $(\Delta t)_c$ are the coherence bandwidth and coherence time of the wireless channel, respectively. The impulse response of such a channel for the kth user can be represented as a tapped delay line model [14] given by

$$h_k(t;\tau) = \sum_{l=0}^{L-1} \beta_{k,l}(t)\delta\left(\tau - \frac{l}{W}\right), \tag{4}$$

where L is the number of paths assumed to be identical for all users, $\delta(t)$ is the Dirac delta impulse function, and $\beta_{k,l}(t)$ is the fading coefficient of user k on the lth path, which has a propagation delay equal to l/W. The fading gain $\beta_{k,l}(t)$ is modeled as a zero mean complex-valued Gaussian random process. Since $T_b \ll (\Delta t)_c$ is assumed, $\beta_{k,l}(t)$ is equal to a constant within the ith symbol interval $iT_b < t < (i+1)T_b$. Moreover, a normalized total energy $\sum_{l=0}^{L-1} E[|\beta_{k,l}(t)|^2] = 1$ is considered. Obviously, from (4), if the receiver cannot resolve and track the fading coefficients on all paths, the performance of a DS-CDMA system will be seriously degraded due to the ISI caused by the multipath effect.

Next, consider multicarrier CDMA systems with the same total bandwidth W, where the signal bandwidth on each sub-carrier is equal to W/M. If the number M of sub-carriers is sufficiently large such that $W/M \ll (\Delta f)_c$, the subdivision of the system bandwidth results in a roughly constant channel frequency response in each sub-carrier. Hence, each sub-channel exhibits a frequency-nonselective effect. From another viewpoint, the ISI due to channel dispersion can be neglected [14], since we have chip duration $M/W \gg T_m$, where $T_m \approx (\Delta f)_c^{-1}$ is the delay spread of the channel. Mathematically, let $s_{k,m}(t)$ and $h_{k,m}(t;\tau)$ be the transmitted signal and channel impulse response, respectively, of user k on the mth sub-carrier, the received signal of a multicarrier CDMA system contributed by the kth user without the effect of noise can be represented as

$$\begin{aligned}
r_k(t) &= \sum_{m=0}^{M-1} s_{k,m}(t) \star h_{k,m}(t;\tau) = \sum_{m=0}^{M-1} \int_{-\infty}^{\infty} S_{k,m}(f) H_{k,m}(t;f) e^{j2\pi f t} df \\
&= \sum_{m=0}^{M-1} H_{k,m}(t;\omega_m/2\pi) s_{k,m}(t) \\
&= \sqrt{\frac{2P_k}{M}} \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} \alpha_{k,m}(i) b_k(i) c_{k,m}(t-iT_b) e^{j\omega_m t},
\end{aligned} \tag{5}$$

where $\star$ denotes the convolution operator, $S_{k,m}(f)$ and $H_{k,m}(t;f)$ the Fourier transform of $s_{k,m}(t)$ and $h_{k,m}(t;\tau)$, respectively, and $\alpha_{k,m}(i) = H_{k,m}(iT_b;\omega_m/2\pi)$ is the constant frequency response of the channel experienced by the mth sub-carrier of user k. Again, we assume $T_b \ll (\Delta t)_c$ such that $H_{k,m}(t;\omega_m/2\pi)$ is a constant over a time period of T_b.

2.3. Received Signal Model

The received signal of a multicarrier CDMA system with K simultaneous users is given by

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t), \tag{6}$$

where $n(t)$ is the complex additive white Gaussian noise (AWGN) with $E[n(t)n^*(s)] = 2N_0\delta(t-s)$. As shown in Figure 2, the received signal is first fed in parallel to M downconverters, correlated with the chip shape waveform $p(t)$, and then sampled at a rate of $1/T_c$, where T_c is the chip interval equal to T_b/N_c. At the mth sub-carrier branch, the signal representation due to the nth chip of symbol index i is given by

$$\begin{aligned}
r_m^{(n)}(i) &= \int_{iT_b+nT_c}^{iT_b+(n+1)T_c} r(t) e^{-j\omega_m t} p^*(t-iT_b-nT_c) dt \\
&= \sum_{k=1}^{K} \sqrt{\frac{2P_k}{M}} \alpha_{k,m}(i) b_k(i) c_{k,m}^{(n)} T_c + n_m^{(n)}(i),
\end{aligned} \tag{7}$$

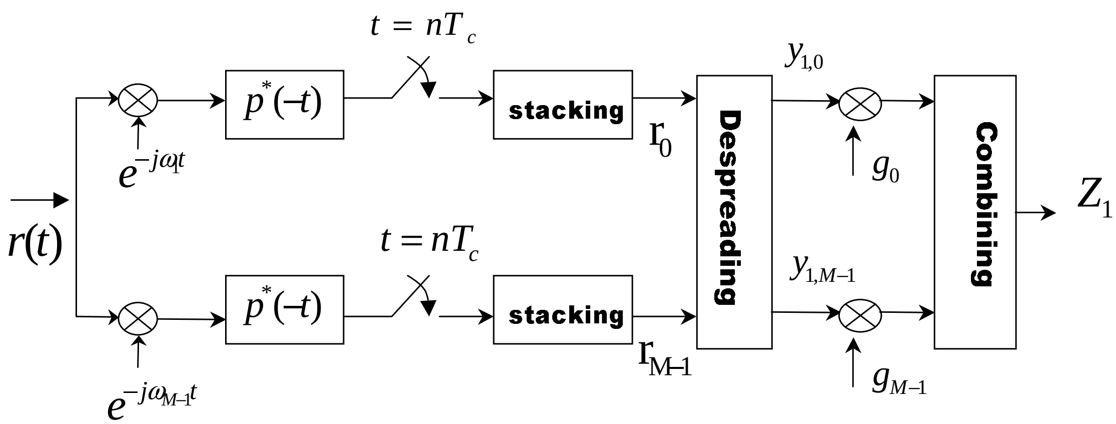

Figure 2. The chip-matched filter and diversity combining receiver model.

where $n_m^{(n)}(i)$ is a complex Gaussian random variable contributed by AWGN

$$n_m^{(n)}(i) = \int_{iT_b+nT_c}^{iT_b+(n+1)T_c} n(t)e^{-j\omega_m t}p^*(t - iT_b - nT_c)dt,$$

with variance $\sigma^2 = 2N_0T_c$. Note that the output of a sub-carrier branch does not contain the contribution from other sub-carriers due to the orthogonality of $e^{j\omega_m t}$ and $e^{j\omega_l t}$, for $m \neq l$. The orthogonal property holds when the fading coefficient is kept unchanged within a chip interval, which is the case for a slowly time-varying channel. The receiver then stacks the outputs until N_c samples have been collected, yielding a vector representation of the received signal. The resulting vector $\mathbf{r_m}$ at the mth sub-carrier due to symbol i can be written as

$$\mathbf{r}_m(i) = \sum_{k=1}^{K} A_k\alpha_{k,m}(i)b_k(i)\mathbf{c}_{k,m} + \mathbf{n}_m(i), \tag{8}$$

where $\mathbf{r}_m(i) = \left[r_m^{(0)}(i), \cdots, r_m^{(N_c-1)}(i)\right]^T$, $\mathbf{c}_{k,m} = \left[c_{k,m}^{(0)}, \cdots, c_{k,m}^{(N_c-1)}\right]^T$ with normalized $|c_{k,m}^{(n)}| = 1/\sqrt{N_c}$ for $n = 0, 1, \cdots, N_c - 1$, and $A_k = \sqrt{2P_k/MT_c}$ accounts for the amplitude scale factor of the kth user, which is the same at all sub-carriers. Note that in an MC-CDMA system, N_c is equal to one, and all the vectors presented in equation (8) are just scalars.

In (7), we discussed the case that the orthogonality holds among sub-carriers when performing chip-matched filtering. However, if the channel is fast time-varying, this property may not be still valid. This happens especially to MC-CDMA due to its large chip interval. To elaborate, since the chip duration T_c is equal to the symbol duration T_b in MC-CDMA, the output of the chip-matched filter at the ith symbol interval of the mth sub-carrier is given by

$$\begin{aligned} r_m(i) &= \int_{iT_b}^{(i+1)T_b} r(t)e^{-j\omega_m t}p^*(t - iT_b)dt \\ &= \sum_{k=1}^{K}\sqrt{\frac{2P_k}{M}}\sum_{r=0}^{M-1} b_k(i)c_k^{(m)}\int_0^{T_b}\alpha_{k,r}(t)e^{-j(\omega_m-\omega_r)t}dt + n_m(i), \end{aligned} \tag{9}$$

where $\alpha_{k,r}(t)$ is a time-varying function of the fading coefficient. It is now time dependent in a finer resolution and cannot be pulled out from the integration. Assuming the fading coefficient changes once during a symbol interval, *i.e.* two fading coefficients $\alpha_{k,m}^L(i)$ and $\alpha_{k,m}^R(i)$ occupying $iT_b \leq t < (i + 1/2)T_b$ and $(i + 1/2)T_b \leq t < (i + 1)T_b$, respectively, the integration in (9) can be evaluated under different situations as

$$\int_0^{T_b}\alpha_{k,r}(t)e^{-j(\omega_m-\omega_r)t}dt = \frac{\alpha_{k,m}^L(i) + \alpha_{k,m}^R(i)}{2}T_b, \qquad \text{when} \quad r = m$$

$$
\begin{aligned}
&= \pm\frac{j}{\pi}\left(\alpha_{k,m\pm1}^{L}(i)-\alpha_{k,m\pm1}^{R}(i)\right)T_b, \qquad \text{when} \quad r-m=\pm1\\
&= 0, \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise,}
\end{aligned}
$$

and $r_m(i)$ can be represented as

$$
r_m(t) = \sum_{k=1}^{K} A_k b_k(i)\left(c_k^{(m-1)}\tilde{\alpha}_{k,m}^{(m-1)} + c_k^{(m)}\tilde{\alpha}_{k,m}^{(m)} + c_k^{(m+1)}\tilde{\alpha}_{k,m}^{(m+1)}\right) + n_m(i), \tag{10}
$$

where

$$
\tilde{\alpha}_{k,m}^{(m-1)} = -j\left(\alpha_{k,m-1}^{L}(i)-\alpha_{k,m-1}^{R}(i)\right)/\pi,
$$

$$
\tilde{\alpha}_{k,m}^{(m)} = \left(\alpha_{k,m}^{L}(i)+\alpha_{k,m}^{R}(i)\right)/2
$$

and

$$
\tilde{\alpha}_{k,m}^{(m+1)} = j\left(\alpha_{k,m+1}^{L}(i)-\alpha_{k,m+1}^{R}(i)\right)/\pi
$$

are contributions of fading gains from the $(m-1)$-th, the m-th and the $(m+1)$-th sub-carriers imposed on the mth sub-carrier respectively. The above discussion shows the ICI effect induced by the time-varying characteristics of the channel in the MC-CDMA system.

3. MRC RECEIVER ANALYSIS

3.1. MRC for Multicarrier CDMA Systems

Based on the received signal model described above, there are numerous design criteria to perform the subsequent signal detection as given in [15]. In this paper, we employ the MRC method to evaluate the performance distinction among single-carrier and multicarrier CDMA systems.

Without loss of generality, let us assume that the first user is the user of interest. Let $y_{1,m}$ denote the scalar after despreading process at the mth sub-carrier, *i.e.*

$$
\begin{aligned}
y_{1,m}(i) &= \mathbf{c}_{1,m}^{H}\mathbf{r}_m(i)\\
&= A_1\alpha_{1,m}(i)b_1(i) + \sum_{k=2}^{K} A_k\alpha_{k,m}(i)\rho_{1,k}b_k(i) + \xi_{1,m}(i),
\end{aligned} \tag{11}
$$

where $\rho_{1,k}$ is the cross-correlation between signature waveforms of users 1 and k, and $\xi_{1,m}(i) = \mathbf{c}_{1,m}^{H}\mathbf{n}_m(i)$ is a Gaussian random variable with zero mean and variance σ^2.

In the sequel, we omit the symbol index i for the sake of clarity. The MRC approach exploits all diversity sources, *i.e.* despread signals $y_{1,m}$ for $m = 0, 1, \cdots, M-1$, and determines the tapped weighting gains g_m's on all sub-carriers by maximizing the instantaneous signal-to-noise ratio (SNR)

$$
SNR \equiv \frac{\left|E\left[b_1^* Z_1|\alpha_1\right]\right|^2}{\mathrm{Var}\left[b_1^* Z_1|\alpha_1\right]}, \tag{12}
$$

of the output decision statistic $Z_1 = \sum_{m=0}^{M-1} g_m y_{1,m}$, where $*$ denotes complex conjugate and $\alpha_1 \equiv \left[\alpha_{1,0}, \cdots, \alpha_{1,M-1}\right]^T$ is the vector of user 1's fading coefficients collected from all sub-carriers. Finally, the decision is based on the sign of the real part in Z_1, *i.e.* $\hat{b}_1 = sgn\left\{\Re(Z_1)\right\}$. In specific, the tapped weight gains are determined by

$$
\{\hat{g}_0, \cdots, \hat{g}_{M-1}\} = \arg\max_{\hat{g}} \frac{\left|E\left[b_1^* \sum_{m=0}^{M-1} g_m y_{1,m}|\alpha_1\right]\right|^2}{\mathrm{Var}\left[b_1^* \sum_{m=0}^{M-1} g_m y_{1,m}|\alpha_1\right]}. \tag{13}
$$

By (11) and defining the interference at the mth sub-carrier $I_m = \sum_{k=2}^{K} A_k \rho_{1,k} \alpha_{k,m} b_k$, the denominator of SNR can be further expanded into

$$\begin{aligned}
\mathrm{Var}\Big[b_1^* \sum_{m=0}^{M-1} g_m y_{1,m} | \alpha_1\Big] &= \mathrm{Var}\Big[\sum_{m=0}^{M-1} g_m \Big(I_m + \xi_{1,m}\Big)\Big] \\
&= \sum_{m=0}^{M-1} \mathrm{Var}\Big[g_m I_m\Big] + \sum_{i \neq j}\sum \mathrm{Cov}\Big[g_i I_i, g_j I_j\Big] + \sum_{m=0}^{M-1} \mathrm{Var}\Big[g_m \xi_{1,m}\Big], \qquad (14)
\end{aligned}$$

where $\mathrm{Cov}\Big[g_i I_i, g_j I_j\Big]$ denotes the covariance between $g_i I_i$ and $g_j I_j$, which can be further evaluated as

$$\begin{aligned}
\mathrm{Cov}\Big[g_i I_i, g_j I_j\Big] &= g_i g_j^* E\Big[I_i I_j^*\Big] \\
&= g_i g_j^* \sum_{k=2}^{K} |A_k|^2 |\rho_{1,k}|^2 E\Big[\alpha_{k,i} \alpha_{k,j}^*\Big]. \qquad (15)
\end{aligned}$$

Note that the righthand side of the first equality in (14) is not conditioned on α_1, because both I_m and $\xi_{1,m}$ are not related to α_1.

It can be seen from (15) that if the correlation between fading coefficients of different sub-carriers are small, the covariance term in (14) diminishes. In such a case, the denominator of (13) can be expressed as

$$\mathrm{Var}\Big[b_1^* \sum_{m=0}^{M-1} g_m y_{1,m} | \alpha_1\Big] \approx \sum_{m=0}^{M-1} \mathrm{Var}\Big[b_1^* g_m y_{1,m} | \alpha_1\Big].$$

Consequently, by employing Cauchy-Schwarz inequality, the maximum value of SNR can be derived as

$$\begin{aligned}
SNR &= \frac{\Big| \sum_{m=0}^{M-1} g_m E\big[b_1^* y_{1,m} | \alpha_1\big] \Big|^2}{\sum_{m=0}^{M-1} |g_m|^2 \mathrm{Var}\Big[b_1^* y_{1,m} | \alpha_1\Big]} \\
&\leq \sum_{m=0}^{M-1} \frac{\Big| E\big[b_1^* y_{1,m} | \alpha_1\big] \Big|^2}{\mathrm{Var}\Big[b_1^* y_{1,m} | \alpha_1\Big]} \\
&= \frac{A_1^2}{\sum_{k=2}^{K-1} A_k^2 |\rho_{1,k}|^2 + \sigma^2} \sum_{m=0}^{M-1} |\alpha_{1,m}|^2, \qquad (16)
\end{aligned}$$

where the equality holds when

$$g_m = \frac{E^*\Big[b_1^* y_{1,m} | \alpha_1\Big]}{\mathrm{Var}\Big[b_1^* y_{1,m} | \alpha_1\Big]} = \frac{A_1 \alpha_{1,m}^*}{\sum_{k=2}^{K-1} A_k^2 |\rho_{1,k}|^2 + \sigma^2}, \quad m = 0, \cdots, M-1. \qquad (17)$$

Note that all factors in (17) independent of m can be dropped without affecting the decision. Therefore, we have

$$\hat{b}_1 = sgn\Big(\Re\big(\sum_{m=0}^{M-1} \alpha_{1,m}^* y_{1,m}\big)\Big). \qquad (18)$$

When there is only one user in the system, the MRC method is optimal in minimizing the bit error probability when all priori probabilities are equally likely [16].

In the DS-CDMA system, the MRC method in association with the RAKE receiver has a decision rule given by [15],

$$\hat{b}_1 = sgn\Big(\Re\big(\sum_{l=0}^{L-1} \beta_{1,l}^* y_l\big)\Big), \qquad (19)$$

where $\beta_{1,l}$ is the fading coefficient of user 1 due to the lth propagation path as defined in (4), and $y_l = \int_0^{T_b} r(t) c_1^*(t - l/W) dt$ is the correlation between the received signal and the delayed spreading sequence of the desired user.

3.2. Performance Analysis

Consider the case of independent binary symbols $\{b_1\}$ with equal priori probability, i.e. $P(b_1 = 1) = P(b_1 = -1) = 0.5$ with uncorrelated fading coefficients between sub-carriers, *i.e.* $E[\alpha_{k,i}\alpha^*_{k,j}] = 0$, for $i \neq j$. Given the decision rule established in (18), the bit error probability (BEP) of the desired user conditioned on its fading coefficients is given by

$$\begin{aligned} P_{e|\alpha_1} &= P\big[\hat{b}_1 = +1|b_1 = -1, \alpha_1\big]P(b_1 = -1) + P\big[\hat{b}_1 = -1|b_1 = +1, \alpha_1\big]P(b_1 = +1) \\ &= P\left[\Re\Big(\sum_{m=0}^{M-1} \alpha^*_{1,m}(I_m + \xi_{1,m})\Big) > A_1 \sum_{m=0}^{M-1} |\alpha_{1,m}|^2 \Big| \alpha_1\right]. \end{aligned}$$

When the total number K of simultaneous users is large, $\Re\big(\sum_{m=0}^{M-1} \alpha^*_{1,m}(I_m + \xi_{1,m})\big)$ converges to a Gaussian random variable by the Central Limit Theorem. Therefore, the BEP of user 1 conditioned on α_1 can be approximated as

$$\begin{aligned} P_{e|\alpha_1} &= Q\left(\sqrt{2SNR_{max}}\right) \\ &= Q\left(\frac{A_1}{\sigma_\dagger}\sqrt{\sum_{m=0}^{M-1} |\alpha_{1,m}|^2}\right), \end{aligned} \tag{20}$$

where SNR_{max} is equal to the righthand side of the third equality in (16), $\sigma^2_\dagger = \frac{1}{2}\left(\sum_{k=2}^{K} A_k^2|\rho_{1,k}|^2 + \sigma^2\right)$, and $Q(x) = \int_x^\infty e^{-y^2/2}/\sqrt{2\pi}dy$ is the Gaussian cumulative function. In order to evaluate the unconditional BEP, we need to average out the effect of $\alpha_{1,m}$ for $m = 0, 1, \cdots, M-1$ inside the $Q(\cdot)$ function. Since $\{\alpha_{1,m}\}$ are independent identically distributed complex Gaussian random variables, $\sum_{m=0}^{M-1} |\alpha_{1,m}|^2$ has a scaled chi-square distribution with $2M$ degrees of freedom. The probability density function (pdf) of a scaled chi-square random variable $X = \sum_{i=1}^{n} X_i^2$, in which $\{X_i\}$ are real-valued Gaussian distributed with zero mean and $\mathrm{Var}(X_i) = \sigma_x^2$, is given by

$$f_X(x) = \frac{1}{\sigma_x^2} f_{\chi^2_n}(\frac{x}{\sigma_x^2}) = \frac{x^{\frac{n}{2}-1} e^{-\frac{x}{2\sigma_x^2}}}{(2\sigma_x^2)^{\frac{n}{2}}\Gamma(\frac{n}{2})}, \qquad x \geq 0,$$

where $f_{\chi^2_n}(y)$ is the pdf of a standard chi-square random variable with degrees of freedom n and $\Gamma(\frac{n}{2}) = \int_0^\infty x^{\frac{n}{2}-1}e^{-x}dx$. For n a positive even integer, we have $\Gamma(\frac{n}{2}) = (\frac{n}{2} - 1)!$. Thus, the pdf of $Y = \sum_{m=0}^{M-1} |\alpha_{1,m}|^2$ can be obtained by setting $n = 2M$ and $\sigma_x^2 = 1/2$, which is

$$f_Y(y) = \frac{y^{M-1}e^{-y}}{\Gamma(M)}, \qquad y \geq 0. \tag{21}$$

Accordingly, the BEP can be computed as

$$\begin{aligned} P_e &= \int_0^\infty Q\left(\frac{A_1}{\sigma_\dagger}\sqrt{y}\right) f_Y(y)dy \\ &= \frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{1+\gamma^{-2}}}\left(1 + \sum_{n=1}^{M-1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n! 2^n (\gamma^2+1)^n}\right), \end{aligned} \tag{22}$$

where the result of the integration can be found in [15] and $\gamma^2 = A_1^2/2\sigma^2_\dagger$. Note that the BEP of a single-carrier DS-CDMA over a frequency non-selective slowly fading channel is given by [15]

$$P_e = \frac{1}{2}\left(1 - \frac{A_1}{\sqrt{\sigma^2 + \sum_{k=1}^{K} A_k^2|\rho_{1,k}|^2}}\right), \tag{23}$$

which is just the case when we set $M = 1$ in (22). By comparing (22) and (23), we see that multicarrier CDMA systems can benefit from the frequency diversity by choosing a large number of sub-carriers, as long as the fading coefficients between them are uncorrelated.

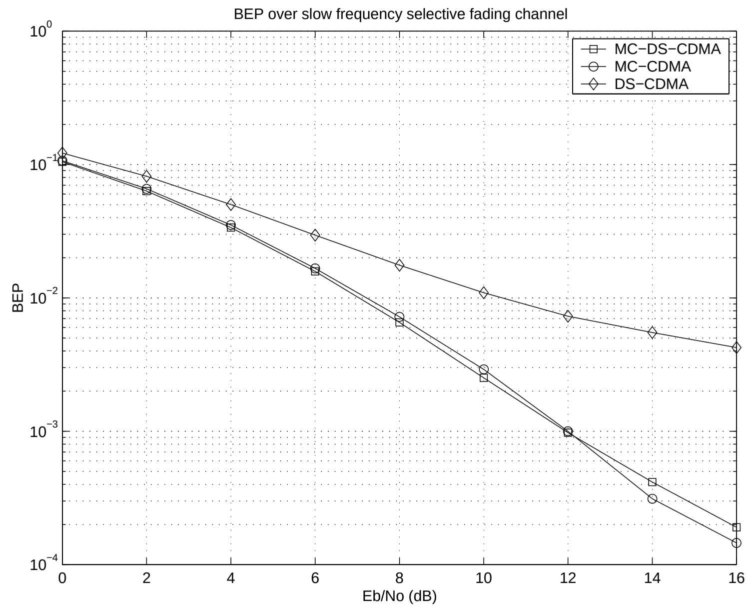

Figure 3. The BEP performance comparison of DS-CDMA, MC-CDMA, and MC-DS-CDMA systems with 5 users and composite spreading ratio 128.

4. SIMULATION

In this section, we present simulation results to illustrate the performance distinction between single-carrier DS-CDMA, MC-CDMA and MC-DS-CDMA systems over two different channel conditions. For these three systems, we employ the MRC method at the receiver and assume perfect channel state information. In order to have a fair comparison, three systems have the same bandwidth and composite spreading ratio $N = MN_c = 128$. It means we have a single-carrier DS-CDMA system with a processing gain N_c of 128, and a MC-CDMA system with $M = 128$ sub-carriers. The number of sub-carriers is designed to be $M = 4$ for MC-DS-CDMA system, yielding a spreading ratio of $N_c = 32$ over each sub-carrier. The simultaneous active users in all the systems are set to be 5. The simulation environment includes: synchronous transmission, coherent detection, binary random signature waveforms and a Rayleigh fading channel corrupted with AWGN. The power levels for all users are fixed and perfectly controlled.

Figure 3 shows the simulation results of a channel, which is frequency-selective and slowly time-varying with four propagation paths and a uniform multipath intensity profile for a DS-CDMA system. Due to the bandwidth subdivision of multicarrier CDMA systems and the assumption that the sub-carrier number is large enough, this channel is frequency non-selective and slowly time-varying for both MC-CDMA and MC-DS-CDMA. Note that, because three systems have the same symbol duration T_b, the slowly time-varying property is experienced by all of them. As can be seen from the figure, the BEP curves with respect to E_b/N_0 for MC-CDMA and MC-DS-CDMA systems are almost identical. This is because that we assume the sub-carrier numbers of both MC-CDMA and MC-DS-CDMA are large enough, resulting in both systems over flat fading channels. The performance of single-carrier DS-CDMA system is degraded by the effect of ISI due to the multipath effect.

In Figure 4, we modify the slowly time-varying property of the channel used in the previous test. All other parameters are kept unchanged. The DS-CDMA system ($N_c = 128$) suffers from a fast time-varying frequency-selective fading channel with 4 propagation paths and the uniform multipath intensity profile. The MC-CDMA ($M = 128$) is transmitted over a frequency-nonselective fast fading channel, with two fading coefficients within a symbol interval, *i.e.* the left and right fading gains occupy during $iT_b \leq t < (i+1/2)T_b$ and $(i+1/2)T_b \leq t < (i+1)T_b$, respectively. The MC-DS-CDMA ($M = 4$, $N_c = 32$) experiences the same channel properties as MC-CDMA except that only one fading coefficient occurs within a chip interval. It can be seen from the figure that MC-DS-CDMA performs the best under this channel condition. This is because the sub-carrier orthogonality still remains for MC-DS-CDMA, while it does not hold for MC-CDMA. As explained in Subsection 2.3, the invalid orthogonal property

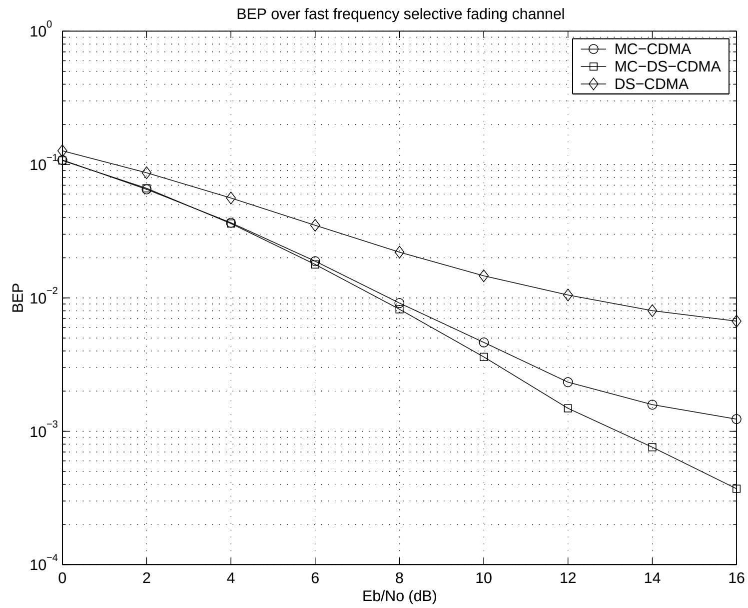

Figure 4. The BEP performance comparison of DS-CDMA, MC-CDMA, and MC-DS-CDMA systems with 5 users and composite spreading ratio 128.

leads to ICI, which deteriorates the performance of MC-CDMA. When E_b/N_0 is small, the AWGN effect dominates such that no clear differentiation between MC-CDMA and MC-DS-CDMA occurs. However, as E_b/N_0 increases, the effect of ICI takes place, the performance of MC-DS-CDMA becomes gradually better. For DS-CDMA, since it suffers from both frequency and time selective fading effects, it has the worst performance.

5. CONCLUSION

In this paper, we compared the performance of DS-CDMA, MC-CDMA and MC-DS-CDMA systems under various fading channel conditions. For the slowly time varying channel, we derived the analytic formula of the BEP for multicarrier CDMA systems, which showed that one can benefit from the frequency diversity gain associated with each sub-channel as long as the fading coefficients on different sub-carriers are uncorrelated. Simulation results showed that multicarrier CDMA systems outperform the DS-CDMA system in its capability to mitigate the multipath effect. For the fast time-varying case, we derived the analytic expression for the received signal of an MC-CDMA system. We showed that its performance is degraded by ICI, if the fading in time is so fast varying that more than one fading coefficients occur within a symbol interval. Simulation results showed that the MC-DS-CDMA system is a preferable choice in this channel due to its shorter chip duration resulting from the larger time-domain spreading ratio.

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